Math 308 Midterm #2, Autumn 2017

Name:

ID#:

Signature:

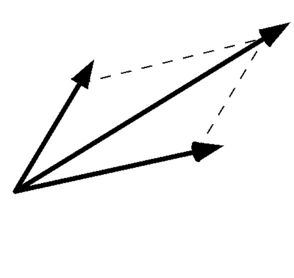
All work on this exam is my own.

Instructions.

- You are allowed a calculator and notesheet (handwritten, two-sided). Hand in your notesheet with your exam.
- Other notes, devices, etc are not allowed.
- Unless the problem says otherwise, **show your work** (including row operations if you row-reduce a matrix) and/or **explain your reasoning**. You may refer to any theorems, facts, etc, from class.
- All the questions can be solved using (at most) simple arithmetic. (If you find yourself doing complicated calculations, there might be an easier solution...)

1	/20
2	/25
3	/20
4	/20
5	/5

Good luck!



(1) (a) Let $A = \begin{bmatrix} 1 & -1 & 4 \\ 0 & 2 & 0 \\ -1 & 1 & -3 \end{bmatrix}$. Compute A^{-1}, showing all work. [10 points]

(b) Let $L \subseteq \mathbb{R}^3$ be the line through the origin spanned by $\vec{\mathbf{v}} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$.

Find linear equations that define L.
(That is, find a system of equations with solution set L.) [10 points]

(2) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the transformation $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$. The matrix A, and an echelon form for A, are given below.

$$A = \begin{bmatrix} 3 & -2 & -1 & 3 \\ -1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 7 \\ 0 & 1 & 2 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Is T is one-to-one? Is T onto? [5 points each]

(b) Give a basis for row(A) and a basis for col(A). [5 points each]

(c) What is nullity(A)? [5 points]

(3) Let $\vec{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the coefficients of a quadratic polynomial, $f(t) = x_1 + x_2 t + x_3 t^2$.

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the function defined by $T(\vec{\mathbf{x}}) =$ the coefficients of $f'(t)$.

For example, $T\left(\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$, because $(1 + 3t + 2t^2)' = 3 + 4t$.

(a) Find a 3×3 matrix A such that $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$. (The entries in A should be numbers. They should not involve t or x_1, x_2, x_3.) [10 pts]

(b) Find a basis for $\ker(T)$. If $\vec{\mathbf{x}} \in \ker(T)$, what does that tell us in terms of the polynomial $f(t)$? (Hint: it's a familiar fact from calculus.) [5 pts]

(c) In terms of the polynomial $f(t)$, what is the meaning of the transformation $S(\vec{\mathbf{x}}) = A^2\vec{\mathbf{x}}$? Explain in a sentence. [5 pts]

(4) Let A, B be $n \times m$ matrices. Let $S \subseteq \mathbb{R}^m$ be the set

$$S = \{\vec{\mathbf{x}} \in \mathbb{R}^m : A\vec{\mathbf{x}} = B\vec{\mathbf{x}}\}.$$

(a) Show that S is a subspace of $\mathbb{R}^m$. [10 pts]
(You may use either the definition, or any theorems or facts from class.)

(b) Suppose $A = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}$. Find a basis for S. [10 pts]
(Hint: S is the set of all $\vec{\mathbf{x}}$ satisfying certain equations.)

(5) (a) Suppose A, B, D are square matrices and $A = B^{-1}DB$.
Simplify A^k to show $A^k = B^{-1}D^kB$, where k is a positive integer.
(If you wish, you can set $k = 3$). [5 pts]

(b) (+3 bonus points)
Let $A = \begin{bmatrix} -2 & -10 \\ 2 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$. **Note**: $A = B^{-1}DB$.

Using the formula in part (a), compute A^{2017}.
(Hint: Note that D is diagonal.)